

Shape Evolution of Nuclei in the Region of ( $A \approx 30$ )
Using Covariant Density Functional Theory

تطورُ أشكالِ الأنوية ذات العدد الكتلى القريب من •r باستخدام نظرية كثافة الدالة

A thesis submitted in partial fulfillment of the requirements for the Masters degree in Physics

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Shape Evolution of Nuclei in the Region of ( $A \approx 30$ ) Using Covariant Density Functional Theory

Accepted by the Faculty Graduated Studies at Birzeit University, Palestine in partial fulfillment of the requirements for the Master Degree in Physics.

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## Declaration

I, Hanaa Mohammad Bashir, declare that this thesis titled, 'Shape Evolution of Nuclei in the region of $(A \approx 30)$ Using Covariant Density Functional Theory' and the work presented in it are my own

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## Dedication

To my beloved parents, brothers and sisters, particularly my eldest sister's family.

To my uncle Othman AbuZarqa, whose belief in me know no bounds!

Thank You!

## Acknowledgements

I wish to thank my committee members for their assistance. A special thanks to Dr. Hazem Abusara, my committee chairman for his support throughout the entire process.


#### Abstract

Shape evolution for $\mathrm{Ne}, \mathrm{Mg}, \mathrm{Si}, \mathrm{S}, \mathrm{Ar}$, and Ca isotopes have even number of neutrons and that are claimed to be in or nearby the $A \approx 30$ mass region of the nuclear chart are studied using covariant density functional theory (CDFT), based on finite range NN-interaction force represented by NL3* and DD-ME2 and zero finite range NN-interaction force represented by DD-PC1. The ground state shape is found to be both oblate and prolate in ${ }^{26} \mathrm{Mg}$ and ${ }^{26} \mathrm{Si}$. The spherical shape is obtained for the Ca isotopes, and for nuclei that have magic neutron number $N=8$ and 20 . The rest of the isotopic chain has only one minimum and alternate between prolate and oblate shapes. Physical properties are calculated at the location of ground-state deformation as the change with neutron number $(\mathrm{N})$ and proton number $(\mathrm{Z})$, such as the binding energy, two-neutron separation energies, proton, neutron and charge radii. In general, a smooth change in these properties is found, except near $N=8$ and 20 one can see a sharp change, which reflects the sudden change in the ground state deformation in the neighboring nuclei. A very good agreement is found with the available experimental data, HF, and FRDM models.


ملخص

في هذهِ الدراسة سنقوُُ بدراسةِ تطور أشكالِ الأنوية ذات العدد الكتلي •r باستخدام نظرية كثافة الدالة وبعض الخصائص النووية كطاقةِ الربط، وطاقة فصل النيوترونات، ونصف قطر كلا من البروتون والنيوترون والشحنة في مستوى الطاقة الأرضي لنظائر النيون والمغنيسيوم والسيليكون والكبريت والأرغون والكالسيو ، بالاعتماد على الكثافة ذات المجال الصفري والنهائي، وبالاعتماد أيضا على التفاعلات غير الخطية للنيوكيونات
 (العدد الذري (Ir=) ونظائر السيليكون (العدد الذري = ع) ونظائر الكبريت (العدد الذري = (Y. = قد تبين أن النواة في مستوى الطاقة الأرضي تأخذ الشكلين المفلطح والمتضخم معاً في نظير المغنيسيوم Y و و نظير السيليكون Y Y ب، وفي تلك الأنوية التي تمتلك عدد
 لها عدد بروتوتات = ب ب وبقية النظائر تتقلبُ بين الشكلين المفلطح والمتضحم. حساباتنا لا تعتمد على اختيار النموذج الذي نجري فيه هذه الحسابات سواء كان نموذج التفاعل غير الخطي أو نموذج التفاعل الكثافي الفعال كما أنَّ جميعَ حساباتِنا تُعطي قيماً معقولةً مقارنة بالقيم العملية والمحسوبة لكل من خصائص مستوى الطاقة الأرضي التي ذكرت

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## Chapter 1

## Introduction

The shape deformation is one of the most fundamental properties of an atomic nucleus, very few nuclei have a spherical shape in their ground state and a variety of shapes can be observed, at a very close energy level[1-5]. Inside the atomic nucleus, protons and neutrons are organized in shells similar to the shells where the electron is organized around the nucleus in the atom. When proton or neutron shells are filled with $2,8,20,28,50,82$, or 126 nucleons, these numbers are called "magic" and nuclei assume spherical shapes. In contrast, nuclei lose their spherical shape and become "deformed" when the number of nucleons is not a magic [6].

Many experimental studies and theoretical approaches to shape deformation have been performed in recent years, such as Coulomb excitation, proton inelastic scattering, self-consistent relativistic mean-field (RMF), Hartree-Fock-Bogolibov (HFB), and macroscopic-microscopic model[7-11].

The shape of the even-even $\mathrm{N}=\mathrm{Z}$ nuclei ${ }^{24} \mathrm{Mg},{ }^{28} \mathrm{Si},{ }^{32} \mathrm{~S},{ }^{36} \mathrm{Ar}$, and ${ }^{40} \mathrm{Ca}$ were studied using axially deformed relativistic mean-field theory by Patra and Praharaj[12]. Large deformations for the ground state have been found for these nuclei.

Patra et al. $[13,14]$ used the axially deformed relativistic mean-field approach to study the entire structural properties of $\mathrm{Ne}, \mathrm{Mg}, \mathrm{Si}, \mathrm{S}, \mathrm{Ar}$ and Ca nuclei. They investigated the nuclei between the neutron dripline and proton dripline, using
three different parameter sets(NL2, NLSH, and TM2). They obtained a very good agreement with the experimental results, especially for the TM2 force. The structure of $\mathrm{Ne}, \mathrm{Na}, \mathrm{Mg}, \mathrm{Al}, \mathrm{Si}$, and S nuclei near the neutron drip-line region is also studied using non-relativistic skyrme Hartree-Fock formalism by the same authors.

Hirata et al.[15] performed a systematic investigation of several light mass nuclei $\left({ }^{24} \mathrm{Mg}\right.$ and $\left.{ }^{40} \mathrm{Ca}\right)$. In his investigation, he used triaxial relativistic mean-field (RMF), and found that the ground state shape is spherical for the ${ }^{40} \mathrm{Ca}$ nucleus and a deformed prolate shape for ${ }^{24} \mathrm{Mg}$. These results are in accordance with the experimental data and the non-relativistic density-dependent Hartree-Fock results.

Similarly, Lalazissis et al. [16] studied the ground-state properties of nuclei with atomic numbers $10 \leq \mathrm{Z} \leq 22$. They used NL-SH parameter set, RMF results provided good agreement with the available empirical data. Several isotopes of Mg , Si and S near the neutron drip line were found to have two ground state minima, oblate and prolate.
Y. Kanada-Enyo[17] studied the deformation of ${ }^{28} \mathrm{Si}$ by focusing on the shape coexistence of the prolate and oblate neutron structure, based on the method of antisymmetrized molecular dynamics. By using the MV1+G3RS force. His result supports an oblate ground-state.

Yao et al. [18] carried out RMF study of Mg isotopes, and found that the triaxiality has no effect on the low-lying states of most Mg isotopes, except for ${ }^{26} \mathrm{Mg}$.

Ying Wang, et al.[19] studied the ground-state properties of even-even nuclei with $Z=10-20$ using the covariant density functional theory and the PC-PK1 parameter set. Their results show that the binding energy of these nuclei under study is not affected by the inclusion of $\gamma$ degree of freedom. However, the authors
have found that triaxiality strongly affects the rotational correction energy. They also studied the effect of the phenomenological collective corrections on the binding energy and the $\mathrm{N}=20$ shell gap. They found that the deviation of the binding energy due to these corrections is reduced from 2.22 MeV to 1.60 MeV and they got a better reproduction of the $\mathrm{N}=20$ shell gap is better .

Dong, G., et al.[20] performed a systematic study on the Ne and Mg isotopes, and their ground state properties using the macroscopic-microscopic model. The authors used isospin-dependent Nilsson potential. They obtained large deformations for the $\mathrm{N}=20$ isotones, ${ }^{30} \mathrm{Ne}$ and ${ }^{32} \mathrm{Mg}$. They found that there exist triaxial deformation (or softness) in the island of inversion. They also showed that for the $\mathrm{N}=18$ and 20 isotones, shape coexistence can be observed with the two shapes being axial, one prolate and the other is oblate.

Density functional theories (DFTs) are extensively used to investigate and understand several nuclear phenomena. This is done using energy density functionals(EDFs) approach, and are approximated by the self-consistent mean-field models. They have been successfully applied in atomic physics[21, 22], where the potential is based on the Coulomb interaction with no phenomenological adjustments, and the system is externally bound. However, in nuclear physics, the situation becomes more complicated since the nucleus is a self-bound system and the spin and isospin degrees of freedom play a crucial part and cannot be ignored[17].

One of the most attractive nuclear DFTs is the covariant density functional theory (CDFT) [23-29] based on the energy density functionals(EDFs), has achieved great success in the description of ground and excited-state properties of both spherical and deformed nuclei throughout the nucleic chart [28, 29].

In this study, we investigate even-even isotopes for a region of the mass number, $A \approx 30$, within the CDFT framework by using three parameter sets: The
density-dependent meson-exchange DD-ME2[26, 30], the density-dependent pointcoupling DD-PC1[31] and the nonlinear meson nucleon coupling NL3*[22]. These parameters provide a good description of different ground states over the entire nuclei chart.

This thesis is organized as follow: In chapter 2, the formulation of the RMF theory for the triaxial deformation will be presented. The results of the investigations of the ground-state properties such as binding energy, two-neutron separation energy and the radii for neutrons and protons are discussed and presented in Section 3. Section 4 is devoted to the summarize the results of the present study.

## Chapter 2

## Formalism

### 2.1 Covariant Density Functional Theory

The Density functional is a tool for a microscopic description of nuclei. It is successful in determining the properties of nuclear ground states such as binding energies, radii, or deformation parameters [32, 33]. Three types of models have been developed to provide a relativistic density functional, the nonlinear meson nucleon coupling model, the density-dependent meson nucleon coupling model, and a density-dependent point coupling model. The main difference between them is the treatment of the range of interaction, mesons, and density dependence. The interaction in the first two classes has a finite range, while the third class uses zero-range interaction [34-37].

A classical relativistic field theory starts from a number of fields $q_{j}(x)$. Their dynamics is determined through a Lagrangian density $\mathcal{L}\left(q, \partial_{\mu} q, t\right)$ and the variational principle [38].

$$
\begin{gather*}
\delta \int \partial^{4} x \mathcal{L}\left(q, \partial_{\mu} q, t\right)=0  \tag{2.1}\\
\partial_{\mu}\left(\frac{\partial L}{\partial\left(\partial_{\mu} q_{j}\right)}\right)-\left(\frac{\partial L}{\partial q_{j}}\right)=0 \tag{2.2}
\end{gather*}
$$

### 2.1.1 The Lagrangian density

from the following Lagrangian density [39, 40]:

$$
\begin{equation*}
\mathcal{L}=\mathcal{L}_{\text {Nucleon }}+\mathcal{L}_{\text {Meson }}+\mathcal{L}_{\text {int }} \tag{2.3}
\end{equation*}
$$

It contains free nucleons described by the Lagrangian density

$$
\begin{equation*}
\mathcal{L}_{\text {Nucleon }}=\bar{\psi}\left(i \gamma_{\mu} \cdot \partial_{\mu}-m\right) \psi \tag{2.4}
\end{equation*}
$$

where $m$ is the bar nucleon mass, and $\psi$ is a Dirac spinor. The Lagrangian for the free mesons contains the following contributions:

$$
\begin{equation*}
\mathcal{L}_{\sigma}=\frac{1}{2}\left(\partial_{\mu} \sigma \partial^{\mu} \sigma-m_{\sigma}^{2} \sigma^{2}\right), \tag{2.5}
\end{equation*}
$$

$$
\begin{equation*}
\mathcal{L}_{\pi}=\frac{1}{2}\left(\partial_{\mu} \pi \partial^{\mu} \pi-m_{\pi}^{2} \pi^{2}\right), \tag{2.6}
\end{equation*}
$$

$$
\begin{equation*}
\mathcal{L}_{\omega}=-\frac{1}{2}\left(\frac{1}{2} \Omega_{\mu \nu} \Omega^{\mu \nu}-m_{\omega}^{2} \omega_{\mu} \omega^{\mu}\right), \tag{2.7}
\end{equation*}
$$

$$
\begin{equation*}
\mathcal{L}_{\rho}=-\frac{1}{2}\left(\frac{1}{2} \vec{R}_{\mu \nu} \vec{R}^{\mu \nu}-m_{\rho}^{2} \rho_{\mu} \rho^{\mu}\right), \tag{2.8}
\end{equation*}
$$

The Lagrangian for photon

$$
\begin{equation*}
\mathcal{L}_{\mathcal{A}}=-\frac{1}{4}\left(\vec{F}_{\mu \nu} \vec{F}^{\mu \nu}\right) \tag{2.9}
\end{equation*}
$$

where $m_{\sigma}, m_{\pi}, m_{\omega}$, and $m_{\rho}$ are the rest masses of mesons, and the $\Omega_{\mu \nu}, \vec{R}_{\mu \nu}$, and $\vec{F}_{\mu \nu}$ are the field tensors given by these equations :

$$
\begin{equation*}
\omega_{\mu \nu}=\partial_{\mu} \Omega_{\nu}-\partial_{\nu} \omega_{\mu} \tag{2.10}
\end{equation*}
$$

$$
\begin{equation*}
\overrightarrow{\mathcal{R}}_{\mu \nu}=\partial_{\mu} \rho_{\nu}-\partial_{\nu} \rho_{\mu}, \tag{2.11}
\end{equation*}
$$

$$
\begin{equation*}
\overrightarrow{\mathcal{F}}_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu} \tag{2.12}
\end{equation*}
$$

The interaction between the nucleons and the mesons described by given Lagrangian

$$
\begin{equation*}
\mathcal{L}_{i n t}=-g_{\sigma} \bar{\psi} \psi \sigma-g_{\omega} \bar{\psi} \gamma_{\mu} \psi \omega_{\mu}-g_{\rho} \bar{\psi} \gamma_{\mu} \vec{\tau} \psi \vec{\rho}_{\mu}-e \bar{\psi} \gamma_{\mu}\left(\frac{1-\tau_{3}}{2}\right) \psi A_{\mu} \tag{2.13}
\end{equation*}
$$

with the coupling constants $g_{\sigma}, g_{\omega}$ and $g_{\rho}$.

### 2.1.2 The meson-exchange model

In the meson exchange model the nucleus is described as a system of pointlike nucleon, Dirac spinors, coupled to mesons and the photons. The nucleons interact by the exchange of several mesons, namely a scalar meson $s$ and three vector particles, $\sigma, \omega, \rho$, and a photon [41, 42]. These mesons are defined by three quantum numbers; spin ( J ), parity $(\mathrm{P})$ and isospin(T). Mesons that participate in this interaction are [35]:

1. The isoscalar scalar $\sigma$ meson, has quantum numbers $(\mathrm{J}=0, \mathrm{~T}=0, \mathrm{P}=1)$, and the corresponding field is a scalar field produce attraction.
2. The isoscalar vector $\omega$ meson, has quantum numbers $(J=1, T=0, P=-1)$, and the corresponding field is a vector field produce the repulsion.
3. The isovector vector $\rho$ meson, has quantum numbers $(\mathrm{J}=1, \mathrm{~T}=1, \mathrm{P}=-1)$, and it couple to the iso vector current.

Starting on a more fundamental level, by introducing a relativistic Lagrangian describing point-like nucleons interacting through the exchange of different types of mesons.

$$
\begin{array}{r}
\mathcal{L}=\bar{\psi}\left(\gamma\left(i \partial_{\mu}-g_{\omega} \omega-g_{\rho} \vec{\rho} \vec{\tau}-e A\right)-m-g_{\sigma} \sigma\right) \psi \\
+\frac{1}{2}(\partial \sigma)^{2}-\frac{1}{2} m_{\sigma}^{2} \sigma^{2}-\frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu}+\frac{1}{2} m_{\omega}^{2} \omega^{2}  \tag{2.14}\\
-\frac{1}{4} \vec{R}_{\mu \nu} \vec{R}^{\mu \nu}+\frac{1}{2} m_{\rho}^{2} \vec{\rho}^{2}-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}
\end{array}
$$

To treat the density dependence in this model Boguta and Bodmer [43] replacing the $\sigma$ mass term by a quartic $\sigma$-potential of the form:

$$
\begin{equation*}
U(\sigma)=\frac{1}{2} m_{\sigma}^{2} \sigma^{2}+\frac{1}{3} g_{2} \sigma^{3}+\frac{1}{4} g_{3} \sigma^{4} \tag{2.15}
\end{equation*}
$$

and the $\omega$ mesons, replacing the mass term by a quadratic a potential of the form:

$$
\begin{equation*}
U\left(\omega_{\mu}\right)=\frac{1}{2} m_{\omega}^{2} \omega^{\mu} \omega_{\mu}+\frac{1}{4} c_{3}\left(\omega^{\mu} \omega_{\mu}\right) \tag{2.16}
\end{equation*}
$$

and the $\rho$ mesons, replacing the mass term by a quadratic a potential of the form:

$$
\begin{equation*}
U\left(\vec{\rho}_{\mu}\right)=\frac{1}{2} m_{\rho}^{2} \vec{\rho}^{\mu} \vec{\rho}_{\mu}+\frac{1}{4} c_{3}\left(\vec{\rho}^{\mu} \vec{\rho}_{\mu}\right) \tag{2.17}
\end{equation*}
$$

The Lagrangian (2.14) contains as parameters the meson masses $m_{\sigma}, m_{\omega}$, and $m_{\rho}$ and the coupling constants $g_{\sigma}, g_{\omega}$, and $g_{\rho}$, and $e$ is the charge of the protons and it vanishes for neutrons.

The density-dependent meson-nucleon coupling model has an explicit density dependence for the meson-nucleon vertices. The coupling constant dependence is defined as:

$$
\begin{equation*}
g_{i}(\rho)=g_{i}\left(\rho_{s a t}\right) f_{i}(x) \tag{2.18}
\end{equation*}
$$

$i$ can be any of the three mesons $\sigma, \omega$, and $\rho$ where the density dependence is given by

$$
\begin{equation*}
f_{i}(x)=a_{i} \frac{1+b_{i}\left(x+d_{i}\right)^{2}}{1+c_{i}\left(x+d_{i}\right)^{2}} \tag{2.19}
\end{equation*}
$$

for $\sigma$ and $\omega$ and by

$$
\begin{equation*}
f_{\rho}(x)=\exp \left(-a_{\rho}(x-1)\right) . \tag{2.20}
\end{equation*}
$$

for the $\rho$ meson.
$x$ is defined as the ratio between the baryonic density $\rho$ at a specific location and the baryonic density at saturation $\rho_{\text {sat }}$ in symmetric nuclear matter. The eight parameters are not independent, but constrained as follows: $f_{i}(1)=1$, $f_{\sigma}^{\prime \prime}(1)=f_{\omega}^{\prime \prime}(1)$, and $f_{i}^{\prime \prime}(0)=0$. These constrains reduce the number of independent parameters for density dependence to three. In our study this model is represented by the parameter sets NL3*, and DD-ME2 given in table 2.1.

Table 2.1: NL3*, DD-ME2, and DD-PC1 parameterizations in RMF Lagrangian

| parameter | NL3* | DD-ME2 | DD-PC1 |
| :---: | :---: | :---: | :---: |
| m | 939 | 939 | 939 |
| $m_{\sigma}$ | 502.5742 | 550.1238 | 0 |
| $g_{\sigma}$ | 10.0944 | 10.5396 | 0 |
| $a_{\sigma}$ | 0.00000 | 1.3881 | -10.04616 |
| $b_{\sigma}$ | 0.00000 | 1.0943 | -9.15042 |
| $c_{\sigma}$ | 0.00000 | 1.7057 | -6.42729 |
| $d_{\sigma}$ | 0.00000 | 0.4421 | 1.37235 |
| $m_{\omega}$ | 782.600 | 783.000 | 0 |
| $g_{\omega}$ | 12.8065 | 13.0189 | 0 |
| $a_{\omega}$ | 0.00000 | 1.3881 | 5.91946 |
| $b_{\omega}$ | 0.00000 | 0.9240 | 8.86370 |
| $c_{\omega}$ | 0.00000 | 1.4620 | 0 |
| $d_{\omega}$ | 0.00000 | 0.4775 | 0.65868 |
| $m_{\rho}$ | 763.000 | 763.000 | 0 |
| $g_{\rho}$ | 4.5748 | 3.6836 | 0 |
| $a_{\rho}$ | 0.00000 | 0.5647 | 0 |
| $b_{\rho}$ | 0.00000 | 0.0000 | 1.83595 |
| $d_{\rho}$ | 0.00000 | 0.0000 | 0.64025 |

### 2.1.3 The point-coupling model

The point-coupling model is another way to construct a relativistic density function. In this model, the mesons exchange replaced by interaction between
the nucleons. Nonlinear point-coupling models have been applied successfully to describe infinite nuclear matter $[46,47]$.

The Lagrangian for the density point coupling model is given by:

$$
\begin{align*}
\mathcal{L} & =\bar{\psi}\left(i \gamma_{\mu} \cdot \partial_{\mu}-m\right) \psi-\frac{1}{2} \alpha_{S}(\hat{\rho})(\bar{\psi} \psi)(\bar{\psi} \psi)-\frac{1}{2} \alpha_{V}(\hat{\rho})\left(\bar{\psi} \gamma^{\mu} \psi\right)\left(\bar{\psi} \gamma_{\mu} \psi\right) \\
& -\frac{1}{2} \alpha_{T} V(\hat{\rho})\left(\bar{\psi} \vec{\tau} \gamma^{\mu} \psi\right)\left(\bar{\psi} \vec{\tau} \gamma_{\mu} \psi\right)-\frac{1}{2} \delta_{S}\left(\partial_{v} \bar{\psi}\right)\left(\partial^{v} \bar{\psi}\right) \\
& -e \bar{\psi} \gamma \cdot A \frac{\left(1-\tau_{3}\right)}{2} \psi \tag{2.21}
\end{align*}
$$

It contains the free-nucleon Lagrangian, the point-coupling interaction terms. The derivative terms accounts for the leading effects of finite-range interaction [26]. This model contains isosclar-scalar, isoscalar-vector, and isovector-vector. It is represented by the DD-PC1 as has been seen in table(2.1).

### 2.2 The Hamiltonian and the equation of motion

From the Lagrangian density in Eq.(2.14), the Hamiltonian operator is:

$$
\begin{equation*}
\mathcal{H}=\int \partial^{3} r\left(\sum_{m} P_{m}-\partial_{t} \phi_{m}-\mathcal{L}(r)\right) \tag{2.22}
\end{equation*}
$$

where, $\phi_{m}=\left(\psi, \sigma, \omega_{\mu}, \overrightarrow{\rho_{\mu}}, A_{\mu}\right)$ and $P_{m}$ is the momentum conjugate operator

$$
\begin{equation*}
\mathcal{P}_{m}=\frac{\partial \mathcal{L}}{\partial\left(\partial \phi_{m} / \partial t\right)} \tag{2.23}
\end{equation*}
$$

The Hamiltonian density of the nucleon - mesons interacting is

$$
\begin{equation*}
\mathcal{H}=\mathcal{H}_{\psi}+\mathcal{H}_{\sigma}+\mathcal{H}_{\omega}+\mathcal{H}_{\rho}+\mathcal{H}_{A}+\mathcal{H}_{i n t} \tag{2.24}
\end{equation*}
$$

where:

$$
\begin{align*}
& \mathcal{H}_{\psi}=\bar{\psi}(\alpha \cdot p+\beta m) \psi,  \tag{2.25}\\
& \mathcal{H}_{\sigma}=-\frac{1}{2} \sigma \Delta \sigma+U_{\sigma}(\sigma),  \tag{2.26}\\
& \mathcal{H}_{\omega}=\frac{1}{2} \omega_{\mu} \omega^{\mu}-U_{\omega}(\omega),  \tag{2.27}\\
& \mathcal{H}_{\rho}=\frac{1}{2} \overrightarrow{\rho_{\mu}} \Delta \overrightarrow{\rho^{\mu}}-U_{\rho}(\rho), \tag{2.28}
\end{align*}
$$

and

$$
\begin{gather*}
\mathcal{H}_{A}=\frac{1}{2} A_{\mu} A^{\mu}  \tag{2.29}\\
\mathcal{H}_{\text {int }}=\left(g_{\sigma} \sigma \bar{\psi} \psi+g_{\omega} \omega_{\mu} \bar{\psi} \gamma^{\mu} \psi+g_{\rho} \overrightarrow{\rho_{\mu}} \bar{\psi} \gamma m a^{\mu} \vec{\tau} \psi+e\left(\frac{1-\tau_{3}}{2}\right) A_{\mu} \bar{\psi} \gamma^{\mu} \psi\right) \tag{2.30}
\end{gather*}
$$

In the Hartree method, the stationary Dirac equation for the nucleons is:

$$
\begin{equation*}
\hat{h}_{D} \psi_{i}=\epsilon_{i} \psi_{i} \tag{2.31}
\end{equation*}
$$

where $\hat{h}_{D}$ is the Hamiltonian of the nucleons with mass $m$

$$
\begin{equation*}
\hat{h}_{D}=\alpha(-i \nabla-V(r))+V_{0}(r)+\beta(m+S(r)) \tag{2.32}
\end{equation*}
$$

the Hamiltonian contains the attractive scalar field $\mathrm{S}(\mathrm{r})$

$$
\begin{equation*}
S(r)=g_{\sigma} \sigma(r) \tag{2.33}
\end{equation*}
$$

and the repulsive time like component of the vector $V_{0}(r)$

$$
\begin{equation*}
V_{0}(r)=g_{\omega} \omega_{0}(r)+g_{\rho} \tau_{3} \rho_{0}(r)+e \frac{1-\tau_{3}}{2} A_{0}(r) \tag{2.34}
\end{equation*}
$$

and the magnetic potential $\mathrm{V}(\mathrm{r})$

$$
\begin{equation*}
V(r)=g_{\omega} \omega(r)+g_{\rho} \tau_{3} \rho(r)+e \frac{1-\tau_{3}}{2} A(r) \tag{2.35}
\end{equation*}
$$

Note that in these eqnarray, the four-vector components of the vector field ( $\omega^{\mu}, \rho^{\mu}, A^{\mu}$ ) are separated into the time-like $\left(\omega_{0}, \rho_{0}, A_{0}\right)$ and the space-like components $[\omega=$ $\left.\left(\omega^{x}, \omega^{y}, \omega^{z}\right), \rho=\left(\rho^{x}, \rho^{y}, \rho^{z}\right), A=\left(A^{x}, A^{y}, A^{z}\right)\right]$.

The corresponding mesons Fields and the electromagnetic field are determined by the Klein-Gordon equations:

$$
\begin{equation*}
\left(-\nabla^{2}+m_{\sigma}^{2}\right) \sigma(r)=-g_{\sigma} \rho_{s}(r)-g_{2} \sigma^{2}(r)-g_{3} \sigma^{3}(r) \tag{2.36}
\end{equation*}
$$

$$
\begin{equation*}
\left(-\nabla^{2}+m_{\omega}^{2}\right) \omega_{0}(r)=g_{\omega} \rho_{\nu} \tag{2.37}
\end{equation*}
$$

$$
\begin{equation*}
\left(-\nabla^{2}+m_{\omega}^{2}\right) \omega_{\mu}(r)=g_{\omega} j_{\mu} \tag{2.38}
\end{equation*}
$$

$$
\begin{equation*}
\left(-\nabla^{2}+m_{\rho}^{2}\right) \rho_{0}(r)=g_{\omega} \rho_{3} \tag{2.39}
\end{equation*}
$$

$$
\begin{equation*}
\left(-\nabla^{2}+m_{\rho}^{2}\right) \vec{\rho}_{\mu}(r)=g_{\rho} \vec{j}_{\mu} \tag{2.40}
\end{equation*}
$$

$$
\begin{equation*}
-\nabla^{2} A_{0}(r)=e \rho_{p}(r) \tag{2.41}
\end{equation*}
$$

$$
\begin{equation*}
-\nabla^{2} A_{\mu}(r)=e \rho_{\mu}^{p}(r) \tag{2.42}
\end{equation*}
$$

with source terms involving the various nucleonic densities and currents

$$
\begin{equation*}
\rho_{s}(r)=\sum_{i=1}^{N} \bar{\psi}_{i}(r) \psi_{i}(r) \tag{2.43}
\end{equation*}
$$

$$
\begin{equation*}
\rho_{\nu}(r)=\sum_{i=1}^{A} \psi_{i}^{+}(r) \psi_{i}(r) \tag{2.44}
\end{equation*}
$$

$$
\begin{equation*}
\rho_{3}(r)=\sum_{i=1}^{A} \psi_{i}^{+}(r) \tau_{3} \psi_{i}(r) \tag{2.45}
\end{equation*}
$$

$$
\begin{equation*}
\rho_{p}(r)=\sum_{i=1}^{A} \psi_{i}^{+}(r)\left(\frac{1-\tau_{3}}{2}\right) \psi_{i}(r) \tag{2.46}
\end{equation*}
$$

$$
\begin{equation*}
j_{\mu}(r)=\sum_{i=1}^{A} \bar{\psi}_{i}(r) \gamma_{\mu} \psi_{i}(r) \tag{2.47}
\end{equation*}
$$

$$
\begin{equation*}
\vec{j}_{\mu}(r)=\sum_{i=1}^{A} \bar{\psi}_{i}(r) \gamma_{\mu} \vec{\tau} \psi_{i}(r) \tag{2.48}
\end{equation*}
$$

In the ground-state solution for an even-even nucleus, spatial vector $A(r)$ is neglected in the calculations, because the coupling constant of the electromagnetic interaction is small compared with the coupling constant of the mesons, and there are no currents (time-reversal invariance) [35].

The components of the vector $\omega$ and $\rho$ mesons lead to the interactions between possible currents. For the $\omega$ meson the interaction is attractive for all combinations ( $p p, n n, p n$ ), and for $\rho$ mesons it is attractive for $p p$ and $n n$ currents but repulsive
for $p n$ currents [36]. The shape coexistence in CDFT framework depends on the spatial components of $\omega$ meson, so there are only two parameters (the mass $m_{\omega}$ and the coupling constant $g_{\omega}$ ) of the $\omega$ meson define the properties of the shape coexistence [Eqs.(2.35, 2.38, 2.40)].

The solution of the CDFT equations corresponds to the ground state of the nucleus it is corresponding to a local minimum in the potential energy surface, so to obtain the solution for any point we used the constrained of quadrupole mass moment. The constrained calculations are performed by imposing constraints on both axial and triaxial mass quadrupole moments [2]. The method of quadratic constraints uses an unrestricted variation of the function

$$
\begin{equation*}
\langle\hat{H}\rangle+\sum_{\mu=0,2} C_{2 \mu}\left(\left\langle\hat{Q}_{2 \mu}\right\rangle-q_{2 \mu}\right)^{2} \tag{2.49}
\end{equation*}
$$

where $\langle\hat{H}\rangle$ is the total energy, $\left(\left\langle\hat{Q_{2 \mu}}\right\rangle\right.$ denotes the expectation values of mass quadrupole operators,

$$
\begin{equation*}
\hat{Q}_{20}=2 z^{2}-x^{2}-y^{2} \quad \text { and } \quad \hat{Q}_{22}=x^{2}-y^{2} \tag{2.50}
\end{equation*}
$$

where: $q_{2 \mu}$ is the constrained value of the multipole moment. $C_{2 \mu}$ is the corresponding stiffness constant. $\sum_{\mu=0,2} \lambda_{\mu} \hat{Q}_{2 \mu}$ is the quadratic constraint adds an extra force term to the system where $\lambda_{\mu}=2 C_{2 \mu}\left(\left\langle\hat{Q}_{2 \mu}\right\rangle-q_{2 \mu}\right)^{2}$

This term is necessary for self consistent solution to force the system to a point in deformation space different from a stationary point. [2]

### 2.3 Pairing correlations

The BCS theory which can accommodate the pairing correlations in the ground states of atomic nuclei are presented [48, 49]. In mean field theory, The relativistic Hartree-Fock-Bogolibov model provides a description to particle-particle (pp) correlation used by pairing field potential $\hat{\Delta}$, and particle-hole (ph) correlation by self consistent field potential.

We take $\mid \phi>$ is the Slater determinate that represents the vacuum with quasiparticle [50], and the $\alpha_{k}, \alpha_{k}^{+}$is the single-nucleons creation and annihilation operator which:

$$
\begin{equation*}
\alpha_{k}=\sum_{n} U_{n k} C_{n}^{+}+V_{n k} C_{n} \tag{2.51}
\end{equation*}
$$

where n is the index refers to original basis, and $\mathrm{U}, \mathrm{V}$ are the Hartree - Bogoluibove wave function determined by variational method.

In the presence of pairing the single-particle density matrix is generalized to two densities: the normal density $\hat{\rho}$ and parity tensor $\hat{K}$

$$
\begin{align*}
& \rho_{n n^{\prime}}=\langle\phi| C_{n}^{+} C_{n} \mid \phi>  \tag{2.52}\\
& \mathcal{K}_{n n^{\prime}}=<\phi\left|C_{n} C_{n}\right| \phi> \tag{2.53}
\end{align*}
$$

The total density functional is :

$$
\begin{equation*}
E_{R H B}=E_{R M F}[\rho]+E_{\text {pair }}[k] \tag{2.54}
\end{equation*}
$$

where

$$
\begin{array}{r}
E_{R H B}\left[\psi, \bar{\psi}, \sigma, \omega^{\mu}, \vec{\rho}^{\mu}, A^{\mu}\right]=\int d^{3} r H(r) \\
E_{R M F}=\sum_{i=1}^{A} \int d^{3} r \psi_{i}^{+}(\alpha p+\beta m)-\frac{1}{2}(\nabla A)^{2}+ \\
\frac{1}{2} e \int d^{3} r j_{p}^{\mu} A_{\mu}+\frac{1}{2} \int d^{3} r\left[\alpha_{s} \rho_{s}^{2}+\alpha_{\nu} j_{\mu} j^{\mu}+\alpha_{T V} \overrightarrow{j_{\mu}} \cdot \overrightarrow{j^{\mu}}+\delta \rho_{s} \rho_{s}\right] \tag{2.56}
\end{array}
$$

and the $E_{\text {pair }}[k]$ is

$$
\begin{equation*}
E_{\text {pair }}[k]=\frac{1}{4} \sum_{n_{1} n_{1}^{\prime}} \sum_{n_{2} n_{2}^{\prime}} K_{n_{1} n_{1}^{\prime}}<n_{1} n_{1}{ }^{\prime}\left|V^{P P}\right| n_{2} n_{2}^{\prime}>K_{n_{2} n_{2}^{\prime}} \tag{2.57}
\end{equation*}
$$

where $<n_{1} n_{1}^{\prime}\left|V^{P P}\right| n_{2} n_{2}^{\prime}>$ is the matrix element of the two body interaction.

$$
\begin{equation*}
V^{p p}\left(r_{1}, r_{2}, r_{1}^{\prime}, r_{2}^{\prime}\right)=-G \delta\left(R-R^{\prime}\right) P(r) P\left(r^{\prime}\right) \tag{2.58}
\end{equation*}
$$

$$
\begin{equation*}
R=\frac{1}{\sqrt{2}}\left(r_{1}+r_{2}\right) \tag{2.59}
\end{equation*}
$$

$$
\begin{equation*}
r=\frac{1}{\sqrt{2}}\left(r_{1}-r_{2}\right) \tag{2.60}
\end{equation*}
$$

$$
\begin{equation*}
P(r)=\left(\frac{1}{4 \pi a^{2}}\right)^{3 / 2} \exp \frac{-r^{2}}{2 a^{2}} \tag{2.61}
\end{equation*}
$$

The RHB-coefficients U and V are obtained by the variational:

$$
\left[\begin{array}{cc}
h_{D}-m-\lambda & \Delta  \tag{2.62}\\
-\Delta^{*} & -h_{D}+m+\lambda
\end{array}\right]\left[\begin{array}{c}
U_{K} \\
V_{K}
\end{array}\right]=E_{K}\left[\begin{array}{c}
U_{K} \\
V_{K}
\end{array}\right]
$$

In (RMFT) the single nucleons has Dirac Hamiltonian $h_{D}$ is a given in Eq.(2.31), $\lambda$ is the chemical potential, $m$ is the mass of nucleons, and $\Delta$ is the pairing field which is :

$$
\begin{equation*}
\Delta_{n_{1} n_{1}^{\prime}}=\frac{1}{2} \sum_{n_{2} n_{2}^{\prime}}<n_{1} n_{1}^{\prime}\left|V^{P P}\right| n_{2} n_{2}^{\prime}>K_{n_{2} n_{2}^{\prime}} \tag{2.63}
\end{equation*}
$$

and

$$
\left[\begin{array}{c}
U_{K}  \tag{2.64}\\
V_{K}
\end{array}\right]
$$

it is an eignvector.

### 2.4 Nuclear shape and deformation

The deformation of the ground state(the nuclear shape), is one of the most fundamental properties of an atomic nucleus, along with its mass and radius. A nucleus may take different shapes varying from spherical to quadrupole (prolate, oblate), and higher order multipole deformations [51].

The instantaneous coordinate $\mathrm{R}(\mathrm{t})$ of a point on the nuclear surface at $(\theta, \phi)$ in terms of the spherical harmonics:

$$
\begin{equation*}
\mathcal{R}(\theta, \phi)=R_{\text {avg }}\left[1+\sum_{\lambda} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi)\right] \tag{2.65}
\end{equation*}
$$

In the first case when $(\lambda=0)$, that gives the monopole, and $\lambda=1$, it is give the dipole deformation, but the important point for our study is when $(\lambda=2)$, which give the quadrupole deformation. For a quadruple-deformed nucleus with elliptical shape we can distinguish a coordinate frame defined by the three axes of deformation. For example we can define the long axis as z, the short axis as
x and the intermediate axis as y (other choices are allowed as well). In this case we have five parameters $\alpha_{\lambda \mu}$, and this five parameters can be reduced to two real parameters $\alpha_{20}, \alpha_{22}$. We defined Hill-Wheeler coordinate in terms of $\alpha_{20}$ and $\alpha_{22}$

$$
\begin{gather*}
\alpha_{20}=\beta \cdot \cos \gamma  \tag{2.66}\\
\alpha_{22}=\frac{1}{\sqrt{2}} \beta \cdot \sin \gamma \tag{2.67}
\end{gather*}
$$

We can connect the quadrupole constraint (2.50) with $\beta, \gamma$

$$
\begin{equation*}
\beta=\sqrt{\frac{4 \pi}{5}} \frac{Q}{r^{2}} \tag{2.68}
\end{equation*}
$$

where

$$
\begin{align*}
& Q=\sqrt{Q_{20}^{2}+Q_{22}^{2}}  \tag{2.69}\\
& \gamma=\tan ^{-1}\left(\frac{Q_{22}}{Q_{20}}\right) \tag{2.70}
\end{align*}
$$

If we substitute Eqs .2.66,2.67 in Eq .2.65, then we obtain :

$$
\begin{equation*}
\mathcal{R}(\theta, \phi)=R_{\text {avg }}\left[1+\beta \sqrt{\frac{5}{16 \pi}}\left(\cos \gamma\left(3 \cos ^{2} \theta-1\right)+\sqrt{3} \sin \gamma \sin ^{2} \theta \cos 2 \phi\right)\right] \tag{2.71}
\end{equation*}
$$

Then we can calculate the increments of the three semi-axes as a function of $\beta$ and $\gamma$

$$
\begin{align*}
& R_{x}=R\left(\frac{\pi}{2}, 0\right)=R_{\text {avg }} \cdot\left[1+\beta \cdot \sqrt{\frac{5}{4 \pi}} \cdot \cos \left(\gamma-\frac{2 \pi}{3}\right)\right]  \tag{2.72}\\
& R_{y}=R\left(\frac{\pi}{2}, \frac{\pi}{2}\right)=R_{\text {avg }} \cdot\left[1+\beta \cdot \sqrt{\frac{5}{4 \pi}} \cdot \cos \left(\gamma+\frac{2 \pi}{3}\right)\right]  \tag{2.73}\\
& \quad R_{z}=R(0,0)=R_{\text {avg }} \cdot\left[1+\beta \cdot \sqrt{\frac{5}{4 \pi}} \cdot \cos (\gamma)\right] \tag{2.74}
\end{align*}
$$

The shape can have axial symmetry, that in the nucleus have an ellipsoid shape and elongated along one of the axis. If it elongated along z -axis it will be prolate axial, and the perpendicular cross section is circular. In the case where the perpendicular
cross section is not circular, then the shape of nucleus will be triaxial[52, 53]. In general if $\gamma$ is a multiple of $60^{\circ}$ then the shape is axial, and when $\gamma$ is not a multiple of 60 it will be triaxial. Thus when $\gamma$ is a multiple of $60^{\circ}$ then the radius along two of the three axis in Eqs. 2.72, 2.73, 2.74 are equal. As we can see :

If $\gamma=0$, the symmetry axis is Z axis, and $R_{x}=R_{y}$.

If $\gamma=60$, the symmetry axis is Y axis, and $R_{x}=R_{z}$.

If $\gamma=120$, the symmetry axis is X axis, and $R_{y}=R_{z}$.

## Chapter 3

## Result and Discussion

In this chapter we present the results of our constrained calculations described in the previous chapter.

The constrained calculations were carried in the $\beta-\gamma$ plane where $\beta$ was taken to be between 0.0 and 0.6 in step of 0.05 , and $\gamma$ in the range of $0^{\circ}, 60^{\circ}$ ] with a step size $\gamma=5^{\circ}$.

Systematic constrained triaxial calculations mapping the quadrupole deformation space defined by $\beta_{2}$ and $\gamma$ have been performed for $\mathrm{A} \approx 30$ isotopes, using DDPC1, NL3*, and DD-ME2 parametrizations. For each isotopic chain, we locate the ground state deformation, and find the value of several physical properties. We also see how the shape of the nucleus in the ground state evolve along the isotopic chain, and how it will effect the physical properties.

In the beginning, we will plot the potential energy surfaces for all the nuclei under consideration using the previously mentioned parameter sets.

### 3.1 POTENTIAL ENERGY SURFACES

### 3.1.1 Ne Isotopes

For Neon isotopes $(Z=10)$ we consider the isotopes with neutron number $N=16$ up to 24. Potential energy surfaces are shown in Figures 3.1,3.2, and 3.3.

At the beginning of the chain we can see traxiality softness and the ground state minimum is flat. However, one can see the existence of a prolate minimum near $\beta=0.15$ in Fig.3.1 for ${ }^{26} \mathrm{Ne}$. As we move along the chain the softness in potential energy surface along the $\gamma$-direction decreases and we can see that the ground state minimum is well localized and moves toward spherical shape. ${ }^{30} \mathrm{Ne}$ is spherical, which is expected since it is a doubly magic nuclei.


Figure 3.1: Potential energy surfaces of even-even Ne isotopes from neutron number $\mathrm{N}=16$ to 24 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.

As the number of neutron increases beyond the magic number and the two additional neutrons are placed in new shell and thus one would expect an increase in the ground state deformation. This in fact what happens and we see that the deformation increases for ${ }^{32,34} \mathrm{Ne}$.


Figure 3.2: Same as Fig. 3.1, but with NL3* parameter set






$\beta_{2}$| ${ }^{32} \mathrm{Ne}$ |
| :--- |
| $\begin{array}{l}16 \\ -14 \\ -12 \\ -10 \\ -8 \\ -6 \\ -4 \\ -2 \\ 0 \\ 0\end{array}$ |



Figure 3.3: Same as Fig. 3.1, but with DD-ME2 parameter set

One can see that there is a slight difference between the results obtained with different parameter sets. This difference only shows up in the lightest isotope due to the triaxiality softness. Unlike DD-PC1, the other parameter sets predicts a spherical shape in the ground state.

The location of the ground state minimum and comparison between the three parameter sets are listed in Table.3.1

TABLE 3.1: Location of the ground state minima indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for (Ne) isotopes using DD-PC1, NL3*, and DD-ME2 parametrization.

| Nucleus | DD-PC1 | NL3 $^{*}$ | DD-ME2) |
| :---: | :---: | :---: | :---: |
| ${ }^{26} \mathrm{Ne}$ | $\left(0.15,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{28} \mathrm{Ne}$ | $\left(0.05,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.1,0^{\circ}\right)$ |
| ${ }^{30} \mathrm{Ne}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{32} \mathrm{Ne}$ | $\left(0.3,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ |
| ${ }^{34} \mathrm{Ne}$ | $\left(0.50,0^{\circ}\right)$ | $\left(0.45,0^{\circ}\right)$ | $\left(0.45,0^{\circ}\right)$ |

### 3.1.2 Mg Isotopes

For Magnesium isotopes $(\mathrm{Z}=12)$ we consider the isotopes with neutron number $\mathrm{N}=10$ up to 22. Potential energy surfaces are shown in Figures 3.4,3.5 and 3.6.
${ }^{22} \mathrm{Mg}$ has two nucleons more than ${ }^{20} \mathrm{Ne}$ which is a doubly magic spherical nuclei. Thus it is well expected that the ground state deformation for ${ }^{22} \mathrm{Mg}$ to be well deformed. In fact it has a prolate ground state minimum with 0.55 as the value of $\beta$ deformation. As the number of neutrons increase as we move along the chain, the value of the deformation decreases.

In addition, a second minimum start to develop on the oblate side. For ${ }^{26} \mathrm{Mg}$ shape coexistence is clearly seen mainly in the DD-PC1 calculations, where it is more pronounced than the other parameter sets. For this nucleus the two minimum has the same value of $\beta$, but one of them is oblate and the other one is prolate. In addition, we also can see that there is softness in the potential energy surface along the $\gamma$-direction. This in fact leads to some kind of discrepancy between the three-parameter set. Any minor change in the occupation of the single-particle states will lead to this small difference among their prediction. This is well seen for ${ }^{30} \mathrm{Mg}$ where NL3* predicts a prolate deformed ground-state, while the other parameter sets predict an oblate shape.

The transition after that from deformed shape for ${ }^{30} \mathrm{Mg}$ to spherical shape for ${ }^{32} \mathrm{Mg}$ is expected due to the magicity of the neutron and proton subsystems.


Figure 3.4: Potential energy surfaces of even-even Mg isotopes from neutron number $\mathrm{N}=10$ to 22 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.

A summary of the results is listed in Table 3.2, and are extracted from Fig.3.4,3.5 and 3.6

Table 3.2: Location of ground state minimum indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for ( Mg ) isotopes using DD-PC1, NL3* and DD-ME2 parameterization.

| Nucleus | DD-PC1 | NL3* | DD-ME2 |
| :---: | :---: | :---: | :---: |
| ${ }^{22} \mathrm{Mg}$ | $\left(0.55,0^{\circ}\right)$ | $\left(0.5,0^{\circ}\right)$ | $\left(0.5,0^{\circ}\right)$ |
| ${ }^{24} \mathrm{Mg}$ | $\left(0.55,0^{\circ}\right)$ | $\left(0.55,0^{\circ}\right)$ | $\left(0.55,0^{\circ}\right)$ |
| ${ }^{26} \mathrm{Mg}$ | $\left(0.3,60^{\circ}\right)$ | $\left(0.3,60^{\circ}\right)$ | $\left(0.3,55^{\circ}\right)$ |
| ${ }^{28} \mathrm{Mg}$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ |
| ${ }^{30} \mathrm{Mg}$ | $\left(0.15,60^{\circ}\right)$ | $\left(0.15,0^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ |
| ${ }^{32} \mathrm{Mg}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{34} \mathrm{Mg}$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ |

All HF calculations [16, 54-56], as well as the FRDM [57], predict the shape of ${ }^{32} \mathrm{Mg}$ to be spherical, which appears as a natural response of the $\mathrm{N}=20$ shell closure.


Figure 3.5: Same as Fig.3.4, but for NL3* parameter set


Figure 3.6: Same as Fig.3.6, but for DD-ME2 parameter set


Figure 3.7: Potential energy surfaces of even-even Si isotopes from neutron number $\mathrm{N}=12$ to 20 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.

We have made further CDFT tests with the three forces used in this study. The results are similar in all cases: the principal minimum is always at $\beta=0$.

### 3.1.3 Si Isotopes

For Silicon isotopes $(Z=14)$ we consider the isotopes with neutron number $N=12$ up to 20. Potential energy surfaces are shown in Figures 3.7,3.8 and 3.9.

For $\mathrm{N}=12$, which corresponds to ${ }^{26} \mathrm{Si}$, there is tow ground state minimum. One of them is oblate and the other one is porlate with very small difference in energy that is less than 0.5 MeV . With the increment of the neutron numbers, one of the minimum disappears and we are left with only one of them.

The ground state of ${ }^{28} \mathrm{Si}$ is experimentally known to be oblate[59], that is in fact in well agreement with the prediction of our calculations. ${ }^{26,28,30,32} \mathrm{Si}$ potential energy surfaces show a deep oblate minimum in all of the three used parameter sets.


Figure 3.8: Same as Fig.3.7, but for NL3* parameter set.


Figure 3.9: Same as Fig.3.7, but for DD-ME2 parameter set.

In table3.3 the location of the ground state minimum for Si isotopes and compared among the three parameter set, and with Skyrme interaction SGII[58]. All of them predict an oblate minima. However, a small variation in the value of $\beta_{2}$ deformation exist when compared with SGII. However, one has to remember that in our calculations the step of $\beta_{3}$ deformation is 0.05 which could be a source of this small deviation
${ }^{26} \mathrm{Si}$ and ${ }^{26} \mathrm{Mg}$ are mirror nuclei of each other, that is the number of proton and neutron in the first one is the same as the number of neutron and proton in the

Table 3.3: Location of the ground state minima indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for (Si) isotopes using DD-PC1, NL3*, DD-ME2 and Skyrme interaction SGII[58] parametrization.

| Nucleus | DD-PC1 | NL3* | DD-ME2 | SGII |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{26} \mathrm{Si}$ | $\left(0.35,60^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,30^{\circ}\right)$ | $\left(0.231,60^{\circ}\right)$ |
| ${ }^{28} \mathrm{Si}$ | $\left(0.35,60^{\circ}\right)$ | $\left(0.35,60^{\circ}\right)$ | $\left(0.35,60^{\circ}\right)$ | $\left(0.276,60^{\circ}\right)$ |
| ${ }^{30} \mathrm{Si}$ | $\left(0.25,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.25,60^{\circ}\right)$ |  |
| ${ }^{32} \mathrm{Si}$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.15,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ |  |
| ${ }^{34} \mathrm{Si}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |  |



Figure 3.10: Potential energy surfaces of ${ }^{26} \mathrm{Si}($ left $),{ }^{26} \mathrm{Mg}$ (right)
second one respectively. Both nuclei in our calculations are found to have the same value of deformation and shape in the ground state.

Fig. 3.10, shows the potential energy surfaces for ${ }^{26} \mathrm{Si}$ and ${ }^{26} \mathrm{Mg}$, respectively. Indeed, the two energy surfaces are similar to each other, and they show the exitance of two ground state minimum, an oblate and a prolate. The energy difference between the oblate and the prolate minima is 0.15 MeV for ${ }^{26} \mathrm{Si}$ and 0.32 MeV for ${ }^{26} \mathrm{Mg}$.

### 3.1.4 S Isotopes

For Sulfur isotopes $(Z=16)$ we consider the isotopes with neutron number $N=8$ up to 16. Potential energy surfaces are shown in Figures 3.11, 3.12, and 3.13.

At the beginning of the chain, $\mathrm{N}=8$, spherical ground state shape appears, as expected. However, as the neutron number increases beyond the magic number one can see that the location of the ground state minima is moving to the right.


Figure 3.11: Potential energy surfaces of even-even $S$ isotopes from neutron number $\mathrm{N}=8$ to 16 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.


Figure 3.12: Same as Fig.3.11, but for NL3* parameter set

The nucleus becomes prolate deformed with quadrupole deformation $\beta=0.20$ for ${ }^{26} S, \beta=0.35$ for ${ }^{28} S$. The addition of two extra neutrons changes the shape from prolate to oblate. Finally, pushing it to be near prolate in ${ }^{32} S$.

A comparison between the three used parameter sets is shown in table3.4. In general a very good agreement is seen except for the case of ${ }^{26} \mathrm{~S}$, where DD-ME2


Figure 3.13: Same as Fig.3.11, but for DD-ME2 parameter set
parameter set predict a spherical shape instead of axially deformed as the other parameter sets.

Table 3.4: Location of the ground state minima indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for (S) isotopes using DD-PC1, NL3* and DD-ME2 parametrization.

| Nucleus | DD-PC1 | NL3* | DD-ME2 |
| :---: | :---: | :---: | :---: |
| ${ }^{24} \mathrm{~S}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{26} \mathrm{~S}$ | $\left(0.2,0^{\circ}\right)$ | $\left(0.15,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{28} \mathrm{~S}$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ | $\left(0.35,0^{\circ}\right)$ |
| ${ }^{30} \mathrm{~S}$ | $\left(0.25,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.25,60^{\circ}\right)$ |
| ${ }^{32} \mathrm{~S}$ | $\left(0.25,0^{\circ}\right)$ | $\left(0.25,0^{\circ}\right)$ | $\left(0.25,0^{\circ}\right)$ |

### 3.1.5 Ar Isotopes

For Argon isotopes $(\mathrm{Z}=18)$ we consider the isotopes with neutron number $\mathrm{N}=16$ up to 24. Potential energy surfaces are shown in Figures 3.14,3.15, and 3.16.

At the beginning of the chain, ${ }^{26} \mathrm{Ar}$ isotope takes a spherical shape, which is expected because of Neutron's magic number $(\mathrm{N}=8)$. As the number of neutrons increases the value of $\beta_{2}$ deformations in the ground state increases and the shape starts to deviate from the spherical shape to be axially deformed (prolate) as seen


Figure 3.14: Potential energy surfaces of even-even Ar isotopes from neutron number $\mathrm{N}=16$ to 24 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.
in ${ }^{28,30} \mathrm{Ar}$ isotopes with $\beta=(0.1,0.2)$. Then it becomes oblate in ${ }^{32,34,36}$ with $\beta=0.2$ for all three parameter sets(see table 3.5). It goes back to spherical shape in ${ }^{38} \mathrm{Ar}$ due to that fact it is a doubly magic nuclei ( has magic numbers in both proton and neutron subsystem)

In addition, we also can see that there is softness in the potential energy surface along the $\gamma$-direction. This, in fact, leads to some kind of discrepancy between the three-parameter set. Any minor change in the occupation of the single-particle states will lead to this small difference among their predictions. This is well seen for ${ }^{40} \mathrm{Ar}$ where DD-ME2 predicts an oblate deformed ground-state, while NL3* predicts a prolate shape with $\beta=0.05$ and spherical shape using DD-PC1. As we reach $\mathrm{A}=42$, we notice that the ground state regains axial(prolate) with $\beta_{2}=0.15$


Figure 3.15: Same as Fig. 3.14, but for NL3* parameter set


Figure 3.16: Same as Fig. 3.14, but for DD-ME2 parameter set
in DD-PC1, but it is oblate using other parameters and still soft in the $\gamma$ direction.
Table 3.5: Location of the first ground state minima indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for (Ar) isotopes using DD-PC1, NL3* and DD-ME2 parameterization.

| Nucleus | DD-PC1 | NL3* | DD-ME2 |
| :---: | :---: | :---: | :---: |
| ${ }^{26} \mathrm{Ar}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{28} \mathrm{Ar}$ | $\left(0.1,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.15,0^{\circ}\right)$ |
| ${ }^{30} \mathrm{Ar}$ | $\left(0.2,0^{\circ}\right)$ | $\left(0.2,0^{\circ}\right)$ | $\left(0.25,0^{\circ}\right)$ |
| ${ }^{32} \mathrm{Ar}$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.15,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ |
| ${ }^{34} \mathrm{Ar}$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ |
| ${ }^{36} \mathrm{Ar}$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ | $\left(0.2,60^{\circ}\right)$ |
| ${ }^{38} \mathrm{Ar}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{40} \mathrm{Ar}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.05,0^{\circ}\right)$ | $\left(0.1,60^{\circ}\right)$ |
| ${ }^{42} \mathrm{Ar}$ | $\left(0.15,00^{\circ}\right)$ | $\left(0.15,60^{\circ}\right)$ | $\left(0.15,60^{\circ}\right)$ |

### 3.1.6 Ca Isotopes

For Calcium isotopes $(\mathrm{Z}=20)$, we consider the isotopes with neutron number N $=14$ up to 24 . Potential energy surfaces are shown in Figures 3.17, 3.18, and 3.19.

The results of deformation parameters $\beta_{2}$ which is listed in the table 3.6, shows that the values of $\beta_{2}$ in all isotopes are used are zeros this is due to the protons magic number $(Z=20)$ (which produce more stability for nucleus).

Table 3.6: Location of the ground state minima indicated by $\left(\beta^{0}, \gamma^{0}\right)$ for (Ca) isotopes using DD-PC1, NL3* and DD-ME2 parameterization.

| Nucleus | DD-PC1 | NL3* | DD-ME2 |
| :---: | :---: | :---: | :---: |
| ${ }^{34} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{36} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{38} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{40} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{42} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |
| ${ }^{44} \mathrm{Ca}$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ | $\left(0.0,0^{\circ}\right)$ |



Figure 3.17: Potential energy surfaces of even-even Ca isotopes from neutron number $\mathrm{N}=14$ to 24 in the $(\beta, \gamma)$ plane as functions of quadrupole deformation, obtained from a triaxial RHB calculations with the DD-PC1 parameter set. The energies are normalized with respect to the binding energy of the absolute minimum. The color scale shown at the right has units of MeV and scaled such that the ground state has a 0 MeV energy.


Figure 3.18: Same as Fig.3.17,but for NL3* parameter set


Figure 3.19: Same as Fig.3.17, but for DD-ME2 parameter set

### 3.2 PHYSICAL PROPERTIES

Shape evolution is coupled to the evolution of several physical properties of the ground state of the atomic nuclei, such as: the binding energy(BE), radii for protons and neutrons, two neutron separation energy $\left(S_{2 n}\right)$ and root mean square charge radii $\left(R_{c}\right)$.

A smooth transition in the ground state deformation along the isotopic chain will be seen as a smooth evolution of these properties. Any sudden jump in the value of the ground state deformation will be reflected as a sharp jump in these properties.

### 3.2.1 Binding Energy

Binding energy is defined as the minimum energy that holds a nucleus together and it is directly related to the stability of nuclei. Figs. 3.20 and 3.21 display the binding energies and the binding energies per nucleon(E/A), respectively.

It may be seen from figure 3.20 that for all the isotopic chains as we increase the neutron numbers the value of the binding energy increases. This can be explained


Figure 3.20: The Binding energies of isotope chains for nuclei: $\mathrm{Ne}, \mathrm{Mg}, \mathrm{Si}, \mathrm{S}, \mathrm{Ar}$ and Ca, obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number. It is compared with the available experimental data[62],[58], FRDM[62], and RMF[62]
by considering light nuclei with small neutron numbers. At the beginning of each chain proton numbers are greater than neutron numbers. Thus the Coulomb repulsion force is large and decreases the stability of the nucleus. The increase in the neutron numbers reduces the repulsion force and add more stability to the atomic nuclei.

The general trend of our three parameter sets used in the calculations is in agreement with each other and with the results obtained from [58, 62]. However, one might notice that there is some kind of deviation in the Ne isotopic chain results.


Figure 3.21: Energy per nucleon of isotope chains for nuclei: Ne, Mg, $\mathrm{Si}, \mathrm{S}, \mathrm{Ar}$ and Ca, obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number, and compared with the available experimental data[62],[58].

Similarly, one notice the same behavior for the binding energy per nucleon plotted in Fig.3.21.

### 3.2.2 Two neutron separation energy $S_{2 n}$

The two-neutron separation energy is an important quantity in analyzing the stabilities of a nucleus. The two-neutron separation energy is defined as:

$$
S_{2 n}=B E_{N, Z}-B E_{N-2, Z}
$$



Figure 3.22: The two-neutron separation energies ( $S_{2 n}$ ), obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number, and compared with the available experimental data[62].

The calculated two-neutron separation energies $S_{2 n}$ of even-even isotopes, as a function of the neutron number N are shown in Fig. 3.22 in comparison with the available experimental data[62]. In Fig. 3.22, it is seen that the results with the density-dependent models reproduce the experimental data quite well. Focusing on the behavior, $S_{2 n}$ gradually increases with N , and a strong abrupt increase is clearly seen at $\mathrm{N}=20$ in experimental and theoretical curves for $\mathrm{Ne}, \mathrm{Mg}, \mathrm{Ar}$ and Ca Nuclei, which indicates the closed-shell at this magic neutron number.

One can also notice a sudden change in the values of $S_{2 n}$ for Si isotopes at $\mathrm{N}=14$, this is due to the fact that ${ }^{26} \mathrm{Si}$ has two ground state minimum while ${ }^{28} \mathrm{Si}$ has one minima.

### 3.2.3 Nuclear Radii

The rms charge, proton and neutron radii are shown in Figs. 3.23,3.24 and 3.25 respectively. The obtained results from all of these models are consistent with each other with very limited variations.

The charge radii shows a strange behavior, for the light isotopes of some the chains in the figures exhibit a higher charge radius as compared to their heavier counterparts. The charge radii assume a minimum value for some intermediatemass nuclei and again increases for higher masses. This effect is well know as the isotopic shift. As the number of proton in the nucleus is larger than the neutron number, the coloumb repuslion force causes the proton to go away from eachother as much as possible. This leads to the increase of the radius. However, as the neutron number increases it decreases the couloumb repuslion force between the protons, and this will reduce the size of the nucleus. But as more neutrons we add the radius starts to increase again due to the increase of the mass number.

The experimental charge radii.[61] of several nuclei for each isotopic chain are shown by solid up triangles in the figures. It can be seen there is a good agreement with the known experimental data. If we compare our results with the relativistic mean-field calculations shown in Fig. 3.22 in Ref.[16], then we will have a very good agreement with the values of the ground state bulk properties.

The charge radius, $R_{c}$, is related to the proton radius, $R_{p}$, by:

$$
\begin{equation*}
R_{c}^{2}=R_{p}^{2}+0.64(\mathrm{fm})^{2} \tag{3.1}
\end{equation*}
$$

where the factor 0.64 is a correction due to the finite size of proton.
we see in Fig. 3.24, that the proton radii are almost equal throughout the isotopic chain, and take the shape of charge radii.


Figure 3.23: The charge $\left(R_{c}\right)$ radii of nuclei obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number. A few available experimental charge radii is shown for comparison.

The neutron radii in Fig. 3.25 shows an increasing trend with neutrons number for all the isotopic chains. For neutron-deficient (proton-rich) nuclei the neutron radius is much smaller than the corresponding charge radius. For the isotopic chain of Ca , the neutron radii show usual parabolic behaviour between $\mathrm{A}=34(\mathrm{~N}$ $=14)$ and $\mathrm{A}=44(\mathrm{~N}=24)$.


Figure 3.24: The proton $\left(R_{p}\right)$ radii of nuclei obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number.


Figure 3.25: The neutron $\left(R_{n}\right)$ radii of nuclei obtained by our DD-PC1, NL3* and DD-ME2 calculations as a function of Neutron Number.

## Chapter 4

## Conclusion

In this thesis, the covariant density functional theory has been employed to investigate the shape deformation and the ground state properties of the even-even ${ }^{26-34} \mathrm{Ne},{ }^{22-34} \mathrm{Mg},{ }^{26-34} \mathrm{Si},{ }^{24-32} \mathrm{~S},{ }^{26-44} \mathrm{Ar}$ and ${ }^{34-44} \mathrm{Ca}$ chain isotopes, using three parametrizations: The density-dependent point-coupling DD-PC1, NL3*, and The density-dependent meson-exchange DD-ME2. Binding energy, two neutron separation energy $S_{2 n}$ and rms radii for (a charge, neutrons, and protons) have been calculated.

The potential energy curves for chain isotopes were plotted as a function of deformation parameter $\beta_{2}$ in triaxial calculations. For Ne, Our calculations establish a deformed prolate shape at ${ }^{26,28,32,34} \mathrm{Ne}$ with deformation $\beta=(.05 \sim .50)$. For Mg , PES predicts there are two minima in ${ }^{26} \mathrm{Mg}$, one of them is oblate and the other one is prolate at, the same appear in ${ }^{26} S i$ at $\beta=0.35$. In the S isotopes, one can see a prolate-oblate transition from ${ }^{28,32} S$. For Ar isotopes, at the beginning of the chain, the ground state has a prolate shape as in ${ }^{28,30} \mathrm{Ar}$, then move to be oblate as in ${ }^{32,34,36,40,42} \mathrm{Ar}$. For Ca isotopes, since the proton number for Ca isotopes is a strong magic number, it leads to spherical nuclei.

The effect of shell closure strongly affect the ground state deformation, it can be seen from our results that spherical magic numbers such as 8 and 20 obey the shell effects and cause the nuclei to have a spherical shape. This effect is observed for several nuclei in our study ${ }^{30} \mathrm{Ne},{ }^{32} \mathrm{Mg},{ }^{34} \mathrm{Si},{ }^{24} \mathrm{~S},{ }^{26,38} \mathrm{Ar}$, and all Ca isotopes.

One can notice that the spherical shape appears in ${ }^{30} \mathrm{Ne},{ }^{32} \mathrm{Mg},{ }^{34} \mathrm{Si},{ }^{24} \mathrm{~S},{ }^{26,38} \mathrm{Ar}$, and all Ca isotopes under study; which proves the deformation parameters decrease when the number of the neutrons be closer to the neutron's magic number $(8,20)$, in other words, nuclei with neutron number (N) far from a magic number are generally deformed. It means that nuclei with magic numbers of neutrons have a "closed shell" that encourages a spherical shape.

The ground-state binding energies and charge radii obtained show good agreement with the data, where available[16,61]. A smooth transition in the ground state deformation will be reflected on the evolution of these properties, and the evolution will be smooth. Any sudden change in the ground state deformation will cause a sharp jump in these properties. A sharp jump in most of physical properties is more apparent above a major close shell, such as at $\mathrm{N}=20$.

The results we obtained are independent of the choice of parametrizations, and consistent with results obtained from other models such as: FRDM[57] and HF[16, 54-56], and with the available experimental data[16, 61].

## Bibliography

[1] K. Heyde, J. L. Wood, Phys. Rev. Mod 83, 1467 (2011).
[2] H. Abusara, S. Ahmad, and S. Othman, Phys. Rev. C 95, 054302 (2017).
[3] K. Nomura, R. Rodríguez-Guzmán, Y. M. Humadi, L. M. Robledo and H. Abusara, Phys. Rev. C 96,034310 (2017).
[4] H. Abusara, S. Ahmad, Phys. Rev. C 96, 064303 (2017)
[5] T. Naz, S. Ahmad, H. Abusara, Acta Physica Polonica B 49(9)1653-1681 (2018).
[6] F. Yang, J. H. Hamilton(1996 Retrieved from http://books.google.com).
[7] P. G. Reinhard, et al, Phys. Rev. C 60, 014316 (1980).
[8] J. L. Egido, et al, Phys. Rev. Lett. 93, 082502 (2004)
[9] Y. Yanagisawa, M. Notani, H. Sakurai, M. Kunibu, H. Akiyoshi, N. Aoi, T. Motobayashi, Nucl. Phys. A 734, 374-377 (2004).
[10] T. Motobayashi, Y. Ikeda, K. Ieki, M. Inoue, N. Iwasa, T. Kikuchi, R. F. Casten, Phys. Lett. B 346(1-2), 9-14 (1995).
[11] H. Iwasakia, T. Motobayashib, H. Sakurai, et al. Phys. Lett. B 620(3-4) 118-124 (2005).
[12] S. K. Patra and C. R. Praharaj, Nucl. Phys. A 565, 442 (1993).
[13] S. K. Patra, R. K. Gupta and W. Greiner, Int. J. Mod. Phys. 06, 04, 641-667 (1997).
[14] S. K. Patra and C. R. Praharaj, arxive:1002.0654vl [nucl-th] (2010).
[15] D. Hirata, K. Sumiyoshi, B. V. Carlson, H. Toki and I. Tanihata, Nucl. Phys. A 609(2), 131-146 (1996).
[16] G. A. Lalazissis, A. R. Farhan, and M. M. Sharma, Nucl. Phys. A 628, 221-254(1998).
[17] Y. Kanada-En'yo Phys. Rev. C 71, 014303 (2005).
[18] J. M. Yao, H. Mei, H. Chen, J. Meng, P. Ring, and D. Vretenar, Phys. Rev. C 83, 014308 (2011).
[19] Y. Wang, J. Li, J. B. Lu, J. M. Yao, Progress of Theoretical and Experimental Physics 2014, Issue 11, November 2014, 113D03.
[20] G. Dong, X. Wang, and S. Yu, Sci. China Phys. Mech. Astron. 58, 112004 (2015).
[21] P. Cejnar, J. Jolie, R. F. Casten, Rev. Mod. Phys. 82, 2155 (2010).
[22] G. A. Lalazissis et al., Phys. Lett. B 671, 36 (2009).
[23] M. Bender, P. H. Heenen, and P. G. Reinhard, Rev. Mod. Phys. 75, 121 (2003).
[24] M. Stoitsov, Phys. Part. Nucl. 41, 868 (2010).
[25] G. A. Lalazissis, P. Ring, and D. Vretenar, (Springer, Berlin, 2004).
[26] T. Niks̆ić, D. Vretenar, P. Ring, Phys. Rev. C 78,034318, (2008).
[27] J. Meng, S. G. Zhou, J. Phys. G 42, 093101 (2015).
[28] J. Meng, H. Toki, S. Zhou, S. Zhang, W. Long, and L. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).
[29] D. Vretenar, A. Afanasjev, G. Lalazissis, and P. Ring, Phys. Rep. 409, 101 (2005).
[30] T. Niks̆ić, N. Paar, D. Vretenar, P. Ring, Comput. Phys. Commun. 185, 1808 (2014).
[31] G. A. Lalazissis, T. Nikšić, D. Vretenar, P. Ring, Phys. Rev. C 71,024312 (2005).
[32] B. D. Serot, J. D. Walecka, Adv. Nucl. Phys. 16 (1986).
[33] R. J. Furnstahl, B. D. Serot, Comments Nucl. Part. Phys. A 2, 23 (2000).
[34] T. Mizutani, F. Myhrer, Phys. Rev. D 35, 1080 (1987).
[35] Y. K. Gambhir, P. Ring, and A. Thimet, ANYALS OF PHYSICS 198, 132179 (1990).
[36] Y. K. Gambhir, and P. Ring, Mod. Phys. Lett. A9, 787-795 (1993).
[37] C. M. Shakin, Wei-Dong Sun, and J. Szweda, Phys. Rev. C52, 3502 (1995).
[38] C. Itzykson and J.B. Zuber; Quantum Field Theory, McGraw Hill, New York (1980)
[39] P. Ring and P. Schuck; The Xuclear Many-body Problem, Springer Verlag, Heidelberg (1980)
[40] Ring, P. Prog. Part. Nucl. Phys., 37, 193âĂŞ263 (1996).
[41] M. Gell-Mann, Phys. Rev. 125, 1067 (1962 ).
[42] S. Barshay, Phys. Rev. 108, 1647 (1957).
[43] Boguta and A.R. Bodmer, Nucl. Phys. A292, 413 (1977)
[44] G. Janssen, K. Holinde, and J. Speth, Phys. Rev. C 49, 2763 (1994).
[45] L. Stodolsky and J. J. Sakurai, Phys. Rev. Lett. 11, 90 (1963).
[46] O. Lourenco, B. M. Santos, M. Dutra, A. Delfino, Phys. Rev. C 94, 045207 (2016).
[47] O. Lourenco, et al, Phys. Rev. C 80, 045206 (2009).
[48] S. Yoshida, and H. Sagawa, Phys. Rev. C 77, 054308 (2008).
[49] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).
[50] K. L. Kowalski, Phys. Rev. D 2, 812 (1970).
[51] P. Moller, et al., arXiv: 1508. 06294 (2015).
[52] N. Tajima, N. Suzuki, Phys. Rev. C 64, 037301 (2001).
[53] R. K. Nesbet, Phys. Rev. 109, 1017 (1958).
[54] N. Tajima, S. Takahara, and N. Onishi, Nucl. Phys. A 603, 23 (1996).
[55] J. Terasaki, H. Flocard, P. H. Heenen, and P. Bonche, Nucl. Phys. A 621, 706,(1997).
[56] J. F. Berger, J. P. Delaroche, M. Girod, S. Peru, J. Libert, and I. Deloncle, 1992, edited by R. Neugart and A. Wohr IOP, Bristol, 1993, p. 487.
[57] P. Möller, J. R. Nix, W. D. Meyers, and W. J. Swiatecki, At. Data Nucl. Data Tables 59, 185 (1995).
[58] M. T. Win, K. Hagino, and T. Koike, Phys. Rev. C 83(1) (2011).
[59] Y. El Bassem, M. Oulne, Nucl, Phys. A 987, 16-28 (2019).
[60] N. Hinohara and Y. Kanada Enayo, arXiv:1008.4444v1.
[61] H. de Vries, C.W. De Jager and C. de Vries, At. Data Nucl. Data Tables 36 (1987) 495.
[62] M. Wang, G. Audi, A. H. Wapstra, et al. Chin. Phys. C 36:1603(2014)


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